Black hole entropy in induced gravity: Reduction to 2D quantum field theory on the horizon

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It is argued that the degrees of freedom responsible for the Bekenstein-Hawking entropy of a black hole in induced gravity are described by two-dimensional quantum field theory defined on the bifurcation surface of the horizon. This result is proved for a class of induced gravity models with scalar, spinor, and vector heavy constituents. [S0556-2821(98)02422-9]

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I. INTRODUCTION

The statistical-mechanical origin of the Bekenstein-Hawking entropy [1,2] is one of the most intriguing problems of black hole physics. There exist several promising approaches to this problem: the string theory approach (see for a review Ref. [3]), calculations of the entropy of some 3D black holes [4,5], an explanation in the framework of loop quantum gravity [6], a mechanism suggested in Sakharov's induced gravity [7], and others. In the induced gravity approach [8–10] the Bekenstein-Hawking entropy is related to the statistical-mechanical entropy of heavy constituent fields which induce the Einstein theory in the low-energy limit. Gravitons in the induced gravity are analogous to phonon excitations in condensed matter systems [11].

A special class of induced gravity models was investigated in Refs. [9,10]. These models contain heavy spinors and scalar constituents propagating in an external gravitational field. The dynamics of the gravitational field arises as the result of quantum effects. The one-loop effective action for quantum constituents gives the low-energy classical action for the Einstein gravity. The constructed models of induced gravity are free from the leading ultraviolet divergences. The induced Newton constant G is completely determined by the parameters of the constituents, and it is finite only if nonminimally coupled scalar fields are present. It was demonstrated that the Bekenstein-Hawking entropy S^{BH} in the induced gravity can be written as

$$S^{BH}=\frac{\mathcal{A}}{4G}=S^{SM}-Q. \tag{1.1}$$

Here $\mathcal{A}$ is the surface area of the horizon, and $S^{SM}=-\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$ is the statistical-mechanical (or entanglement) entropy of the thermally excited (with thermal density matrix $\hat{\rho}$) constituent fields propagating near the horizon [26]. The quantity

$$Q=\sum_s \xi_s \int_\Sigma \langle \hat{\phi}_s{}^2\rangle \tag{1.2}$$

is the sum of contributions of the nonminimally coupled scalar fields $\hat{\phi}_s$. In this relation ξ_s are parameters of nonminimal coupling and $\langle \hat{\phi}_s{}^2\rangle$ is the quantum average of the squares of the scalar operators on the bifurcation surface Σ. In these particular models the origin of Q is related to the nonminimal couplings of the scalar fields. It was shown in [10], that Q can be interpreted as a Noether charge [12–14] and it determines the difference between the energy of the fields and their canonical Hamiltonian.

The subtraction in Eq. (1.1) is unavoidable for the following reasons. The contribution of each (Bose and Fermi) constituent field into S^{SM} is positive and divergent. Thus, the entropy S^{SM} is divergent, while black hole entropy S^{BH} is finite. In formula (1.1) the divergence of S^{SM} is exactly compensated by the divergence of the quantity Q.

There is a more profound reason why the Noether charge Q appears in Eq. (1.1). The Bekenstein-Hawking entropy S^{BH} determines the degeneracy of states of a black hole. It was argued in [10] that this degeneracy can be calculated by counting states of constituents with fixed total *energy*. On the other hand, the entropy S^{SM} is directly related to the distribution over the levels of the *Hamiltonian* of constituent fields. The additional term Q is required to relate it to the distribution over the energy levels.

In the present work we consider a wider class of induced gravity models which besides scalar and spinor constituents contains also massive vector fields. For briefness we call such models *vector models*. We demonstrate that the parameters of vector models can be chosen to exclude the leading ultraviolet divergences even if all scalar fields are minimally coupled. The remarkable fact is that the relation (1.1) is still valid. The Noether charge Q in Eq. (1.1) is related now to the ''natural'' coupling of vector fields with the curvature. The universality of the form of Eq. (1.1) seems to be a quite general property of the induced gravity theories.

The important property of a vector model is that its only free parameters are the masses of the fields, while the ''nonminimal couplings'' are fixed by the form of the action of the vector fields. As we will see, this property makes possible a

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new, interesting interpretation of the Bekenstein-Hawking entropy in induced gravity in terms of a two-dimensional quantum theory on Σ. Thus, induced gravity models provide a simple realization of the holographic principle: the black hole entropy is encoded in ''surface'' degrees of freedom, i.e., in the degrees of freedom of the theory which propagate very close to the black hole horizon. The holographic principle was formulated in [15,16] (see also the recent paper in [17]) and at the present moment it is actively discussed in the framework of string theory [18–21].

This paper is organized as follows. In Sec. II we describe the models of induced gravity with vector fields. Section III is devoted to the derivation of Eq. (1.1) for these models. Special attention here is paid to the calculation of the statistical-mechanical entropy of vector fields in the presence of the Killing horizon and to the properties of the Noether charge which is connected with nonminimal vector couplings. These results enable us to adopt a statistical-mechanical explanation of the Bekenstein-Hawking entropy given in Ref. [10] to a more general class of induced gravity models. In Sec. IV we establish the relation between the Bekenstein-Hawking entropy and the effective action of a 2D free massive quantum field ''living'' on the bifurcation surface Σ of the horizons. As we show this relation is satisfied for induced gravity obtained from a theory with partly broken supersymmetry. Concluding remarks and a brief discussion of the holographic property of the black hole entropy in induced gravity theories are presented in Sec. V. The relation between the energy, the Hamiltonian, and the Noether charge for massive vector fields is derived in the Appendix.

We use the sign conventions of the book in [22] and, thus, we work with the signature $(-+++)$ for the Lorentzian metric.

II. INDUCED GRAVITY MODELS WITH VECTOR FIELDS

The vector model[1] consists of N_s minimally coupled scalar fields ϕ_i with masses $m_{s,i}$, N_d spinors ψ_j with masses $m_{d,j}$, and N_v vector fields V_k with masses $m_{v,k}$. The classical actions of the fields are standard,

$$I_s[\phi_i]=-\frac{1}{2}\int dV[(\nabla\phi_i)^2+m_{s,i}^2\phi_i^2], \tag{2.1}$$

$$I_d[\psi_j]=\int dV\overline{\psi}_j(\gamma^\mu\nabla_\mu+m_{d,j})\psi_j, \tag{2.2}$$

$$I_v[V_k]=-\int dV\left[\frac{1}{4}F_k^{\mu\nu}F_{k\mu\nu}+\frac{1}{2}m_{v,k}^2V_k^\mu V_{k\mu}\right], \tag{2.3}$$

where $dV=\sqrt{-g}d^4x$ is the volume element of 4D spacetime $\mathcal{M}$ and $F_{k\mu\nu}=\nabla_\mu V_{k\nu}-\nabla_\nu V_{k\mu}$. The corresponding quantum effective action of the model is

$$\Gamma=\sum_{i=1}^{N_s}\Gamma_s(m_{s,i})+\sum_{j=1}^{N_d}\Gamma(m_{d,j})+\sum_{k=1}^{N_v}\Gamma(m_{v,k}). \tag{2.4}$$

Γ is a functional of the metric $g_{\mu\nu}$ of the background spacetime. The scalar and spinor actions follow immediately from Eqs. (2.1) and (2.2):

$$\Gamma_s(m_{s,i})=\tfrac{1}{2}\log\det(-\nabla^2+m_{s,i}^2), \tag{2.5}$$

$$\Gamma_d(m_{d,j})=-\log\det(\gamma^\mu\nabla_\mu+m_{d,j}). \tag{2.6}$$

As a result of the equation of motion, a massive vector field V_μ obeys the condition $\nabla^\mu V_\mu=0$, which leaves only three independent components. Under quantization this condition can be realized as a constraint so that the effective action for vector fields takes the form

$$\Gamma_v(m_{v,k})=\tilde{\Gamma}_v(m_{v,k})-\Gamma_s(m_{v,k}), \tag{2.7}$$

$$\tilde{\Gamma}_v(m_{v,k})=\tfrac{1}{2}\log\det(-\nabla^2\delta_\nu^\mu+R_\nu^\mu+m_{v,k}^2\delta_\nu^\mu), \tag{2.8}$$

where R_ν^μ is the Ricci tensor. The functional $\tilde{\Gamma}_v(m_{v,k})$ represents the effective action for a massive vector field which we will denote as $A_{k,\mu}$. The classical action for $A_{k,\mu}$ which results in Eq. (2.8) is

$$\tilde{I}_v[A_k]=-\frac{1}{2}\int dV[\nabla^\mu A_k^\nu\nabla_\mu A_{k\nu}+R_{\mu\nu}A_k^\mu A_k^\nu+m_{v,k}^2A_k^\mu A_{k\mu}]. \tag{2.9}$$

The field A_k^μ obeys no constraints and carries an extra degree of freedom. The contribution of this unphysical degree of freedom in Eq. (2.8) is compensated for by subtracting the action $\Gamma_s(m_{v,k})$ of a scalar field with the mass $m_{v,k}$; see Eq. (2.7).

In general, the effective action (2.4) is an ultraviolet-divergent quantity. Let us discuss now the constraints which have to be imposed on the masses of the constituents to eliminate the leading divergences in Γ. The divergences related to each particular field follow from the Schwinger-DeWitt representation

$$\Gamma_i=-\frac{\eta_i}{2}\int_\delta^\infty\frac{ds}{s}e^{-m_i^2s}\mathrm{Tr}\,e^{-s\Delta_i}, \tag{2.10}$$

where $\eta_i=+1$ for Bose fields and -1 for Fermi fields, and δ is an ultraviolet cutoff. The divergences come from the lower integration limit where one can use the asymptotic expansion of the trace of the heat kernel operator of Δ_i:

$$\mathrm{Tr}\,e^{-s\Delta_i}\simeq\frac{1}{(4\pi s)^2}\int dV(a_{i,0}+sa_{i,1}+\cdots). \tag{2.11}$$

For the fields under consideration we have

$$\Delta_s=-\nabla^\mu\nabla_\mu,\quad a_{s,0}=1,\quad a_{s,1}=\frac{1}{6}R, \tag{2.12}$$

[1]A similar model of induced gravity was discussed in [23].

$$\Delta_d=-(\gamma^\mu\nabla_\mu)^2,\quad a_{d,0}=4,\quad a_{d,1}=-\frac{1}{3}R,\tag{2.13}$$

$$(\Delta_v)^\mu_\nu=-\nabla^\rho\nabla_\rho\delta^\mu_\nu+R^\mu_\nu,\quad a_{v,0}=4,\quad a_{v,1}=-\frac{1}{3}R.\tag{2.14}$$

As in the case of the model considered in Ref. [9], we require a vanishing of the cosmological constant and a cancellation of the divergences of the induced Newton constant. These conditions can be written down with the help of the following two functions:

$$p(z)=\sum_{i=1}^{N_s} m_{s,i}^{2z}-4\sum_{j=1}^{N_d} m_{d,j}^{2z}+3\sum_{k=1}^{N_v} m_{v,k}^{2z},$$
$$q(z)=\sum_{i=1}^{N_s} m_{s,i}^{2z}+2\sum_{j=1}^{N_d} m_{d,j}^{2z}-3\sum_{k=1}^{N_v} m_{v,k}^{2z}.\tag{2.15}$$

As can be shown by using Eqs. (2.4), (2.10)–(2.14), the induced cosmological constant vanishes when

$$p(0)=p(1)=p(2)=p'(2)=0.\tag{2.16}$$

The induced Newton constant G is finite if

$$q(0)=q(1)=0.\tag{2.17}$$

The constraints result in simple relations

$$N_s=N_d=N_v,\quad \sum_{i=1}^{N_s} m_{s,i}^2=\sum_{j=1}^{N_d} m_{d,j}^2=\sum_{k=1}^{N_v} m_{v,k}^2.\tag{2.18}$$

They show that one cannot construct a theory with finite cosmological and Newton constants from vector and spinor fields only.

The low-energy limit of the theory corresponds to the regime when the curvature radius L of the spacetime $\mathcal{M}$ is much greater than the Planck length m_{Pl}^{-1}. In this limit the effective action Γ of the theory can be expanded in the curvature. The terms in this series are local and the leading terms can be calculated explicitly. In the linear in curvature approximation Γ coincides with the Einstein action[2]

$$\Gamma[g]\simeq\frac{1}{16\pi G}\left(\int_{\mathcal{M}}dVR+2\int_{\partial\mathcal{M}}dvK\right).\tag{2.19}$$

Here dv is the volume element of $\partial\mathcal{M}$. The Newton constant is determined by the following expression:

$$\frac{1}{G}=\frac{1}{12\pi}q'(1)=\frac{1}{12\pi}\sum_{i=1}^{N}(m_{s,i}^2\ln m_{s,i}^2+2m_{d,i}^2\ln m_{d,i}^2-3m_{v,i}^2\ln m_{v,i}^2).\tag{2.20}$$

Here, according to Eqs. (2.18), we put $N=N_s=N_d=N_v$. From this expression it is easy to conclude that at least some of the constituents must be heavy and have mass comparable with the Planck mass m_{Pl}. For simplicity in what follows we assume that all the constituents are heavy.

Let us analyze models where conditions (2.15) and (2.16) are satisfied. Equations (2.18) are trivially satisfied when all fields are in supersymmetric multiplets. However, in such supersymmetric models $p(z)=q(z)\equiv 0$ (because masses of the fields in the same supermultiplet coincide) and the induced gravitational constant vanishes. A nontrivial induced gravity theory can be obtained if the supersymmetry is partly broken by splitting the masses of the fields in the supermultiplets.

Let us demonstrate this by an example. Consider a model with N massive supermultiplets. Each multiplet consists of one scalar, one Dirac spinor, and one vector field, so that the numbers of Bose and Fermi degrees of freedom coincide.[3] We suggest that masses of vector and spinor fields are equal, $m_{v,i}=m_{d,i}\equiv m_i$ (here i is the number of the multiplet). The masses of the scalar partners are assumed to be $m_{s,i}=(1+x_i)m_i$, where x_i is a dimensionless coefficient. The case when $|x_i|\ll 1$ corresponds to slightly broken supersymmetry. For this case,

$$p(z)=q(z)=\sum_{i=1}^{N} m_i^{2z}[(1+x_i)^{2z}-1]\simeq 2z\sum_{i=1}^{N} x_i m_i^{2z}.\tag{2.21}$$

Now Eqs. (2.16), (2.17), and (2.20) take the simple form

$$\sum_{i=1}^{N} x_i m_i^2=0,\quad \sum_{i=1}^{N} x_i m_i^4=0,\tag{2.22}$$

$$\sum_{i=1}^{N} x_i m_i^4\ln m_i^2=0,\quad \frac{1}{G}\simeq\frac{1}{6\pi}\sum_{i=1}^{N} x_i m_i^2\ln m_i^2.\tag{2.23}$$

This is a system of linear equations for x_i which for $N\geq 4$ has nontrivial solutions.

The induced gravity constraints provide a cancellation of the leading ultraviolet divergences. However, some logarithmical divergences are still present on general backgrounds. On the Schwarzschild background the logarithmic diver-

[2]To induce the correct boundary term in Eq. (2.19) one has to add to Γ an integral of averages of field operators on the spatial boundary $\partial\mathcal{M}$; see [27]. These terms are not relevant for our analysis. Let us emphasize that we are interested in the statistical-mechanical computation of the black hole entropy for which only the region near the horizon is important.

[3]Supersymmetric models with free massive scalar, spinor, and vector fields are discussed, for instance, in Ref. [28].

gences are pure topological and can be neglected. That is why in what follows we restrict the analysis to black holes of this type.[4]

III. STATISTICAL CALCULATION OF THE BLACK HOLE ENTROPY

Let us now calculate the statistical-mechanical entropy S^{SM} in the vector models of induced gravity and compare it with the Bekenstein-Hawking entropy of a black hole. As a result of this comparison, we prove the validity of Eq. (1.1) for these models.

The canonical ensemble of constituent fields on a static, asymptotically flat background can be described by standard methods. The statistical-mechanical entropy of the fields is determined from the free energy

$$F(\beta)=-\beta^{-1}\ln \mathrm{Tr}\exp(-\beta{:}\hat{H}{:})$$
$$=\eta\beta^{-1}\int_0^\infty d\omega\frac{dn}{d\omega}\ln(1-\eta e^{-\beta\omega}). \qquad (3.1)$$

Here β is the inverse temperature measured at infinity and ${:}\hat{H}{:}$ is the Hamiltonian of the system which is defined as the generator of canonical transformations along Killing time. The factor $\eta=1$ for bosons and $\eta=-1$ for fermions, ω are the frequencies of single-particle excitations, and $dn/d\omega$ is the density of levels ω.

When the background space-time is the exterior region of a black hole the single-particle spectra have a number of specific properties because of the presence of the Killing horizons [24]. In particular, the density of states $dn/d\omega$ infinitely grows near the horizon. Although this divergence has an infrared origin, regularizations of the ultraviolet type can be applied to make $dn/d\omega$ finite. For scalar and spinor fields on general static backgrounds the divergences of $dn/d\omega$ were computed in [25]. In the Pauli-Villars regularization the leading divergences for scalar and Dirac spinor fields of the mass m are

$$\frac{dn_s(m)}{d\omega}=\frac{b(m)}{8\pi^2\kappa}\mathcal{A},\qquad \frac{dn_d(m)}{d\omega}=\frac{b(m)}{2\pi^2\kappa}\mathcal{A}, \qquad (3.2)$$

$$b(m)=c\mu^2-m^2\ln\frac{\mu^2}{m^2}. \qquad (3.3)$$

Here $\kappa=(4M)^{-1}$ is the surface gravity of the black hole, μ is the Pauli-Villars cutoff, and $c=\ln\frac{729}{256}>0$.

Modes propagating in the vicinity of the horizon give the main contribution to the densities of levels. That is why the quantity $dn/d\omega$ scales as the surface area $\mathcal{A}$ of the horizon. This also means that to get Eqs. (3.2) it is sufficient to restrict oneself to the Rindler approximation of the black hole metric:

$$ds^2=-\kappa^2\rho^2dt^2+d\rho^2+dz_1^2+dz_2^2. \qquad (3.4)$$

Here $\rho>0$, and t is the Rindler time coordinate. In this approximation the densities of levels for high-spin fields can be computed by using expression (3.2) for scalars and spinors.

Let us consider a massive vector field in Minkowski spacetime. We denote by X^m $(m=0,\ldots,4)$ the Cartesian coordinates in this space and by $V_m=(V_0,V_a)$, $a=1,2,3$, the components of the vector field with respect to the Cartesian frame. Then the equations of motion which extremize vector field action (2.3) are simply a set of Klein-Gordon equations for four ''scalars'' V_m plus the additional constraint $\partial_m V^m=0$. The constraint serves to express the time component V_0 in terms of other components V_a. The contribution of this component to the energy is negative and V_0 cannot be considered as an independent physical degree of freedom. The density of levels of the vector field $dn_v/d\omega$ multiplied by $d\omega$ is the number of independent solutions $V_m(t,\rho,z)=e^{-i\omega t}V_m(\rho,z)$ of the field equations with frequencies in the interval $(\omega,\omega+d\omega)$. The solutions are determined by three independent functions V_α. Therefore, in the Rindler approximation $dn_v/d\omega$ is greater by a factor of 3 than the density of levels of a scalar field of the same mass m. From Eqs. (3.2) we find

$$\frac{dn_v(m)}{d\omega}=3\frac{dn_s(m)}{d\omega}=\frac{3b(m)}{8\pi^2\kappa}\mathcal{A}. \qquad (3.5)$$

The curvature corrections may change this relation but they are not important for further analysis.

The statistical-mechanical entropy

$$S=\beta^2\frac{\partial F}{\partial\beta}, \qquad (3.6)$$

of scalar, spinor, and vector fields, follows from Eqs. (3.1), (3.2), and (3.5):

$$S_s(m_{s,i})=\frac{b(m_{s,i})}{48\pi}\mathcal{A},$$

$$S_d(m_{d,i})=\frac{2b(m_{d,i})}{48\pi}\mathcal{A},$$

$$S_v(m_{v,i})=\frac{3b(m_{v,i})}{48\pi}\mathcal{A}. \qquad (3.7)$$

Expressions (3.7) are obtained from formula (3.6) at the Hawking temperature, i.e., at $\beta=2\pi/\kappa=8\pi M$. The statistical-mechanical entropy of the constituents in the induced gravity model is

$$S^{SM}=\sum_{i=1}^{N}\left[S_s(m_{s,i})+S_d(m_{d,i})+S_v(m_{v,i})\right]. \qquad (3.8)$$

[4]At least some of the logarithmic divergences can be eliminated in more complicated models, for instance, in models which contain both vector and nonminimally coupled scalar fields. These models allow one to generalize the analysis of the black hole entropy problem in induced gravity to charged black holes.

By substituting Eqs. (3.7) into Eq. (3.8) and taking into account Eqs. (3.3), (2.18) we get

$$S^{SM}=\frac{1}{48\pi}\sum_{i=1}^{N}[m_{s,i}^2\ln m_{s,i}^2+2m_{d,i}^2\ln m_{d,i}^2+3m_{v,i}^2\ln m_{v,i}^2]\mathcal{A}+\frac{1}{8\pi}\left[cN\mu^2-\ln\mu^2\sum_{i=1}^{N}m_{v,i}^2\right]\mathcal{A}. \tag{3.9}$$

Let us now calculate the Noether charge Q for our model. It is instructive to discuss first the entropy of a black hole in a classical theory. According to Wald and other authors [12–14], the black hole entropy can be interpreted as a Noether charge and obtained from the Lagrangian L of the theory. For theories which do not include the derivatives of a metric higher than second order the entropy can be written in the form

$$S=-8\pi\int_{\Sigma}t_{\mu}n_{\nu}t_{\lambda}n_{\rho}\frac{\partial L}{\partial R_{\mu\nu\lambda\rho}}d\sigma, \tag{3.10}$$

where $R_{\mu\nu\lambda\rho}$ is the Riemann tensor. The integration in Eq. (3.10) goes over the bifurcation surface Σ of the horizon, and $d\sigma$ is the volume element of Σ ($\int_{\Sigma}d\sigma=\mathcal{A}$). Vectors t_{μ} and n_{μ} are two mutually orthogonal vectors normal to Σ such that $t^2=-1$ and $n^2=1$.

For the Einstein theory, Eq. (3.10) reproduces the Bekenstein-Hawking formula for the black hole entropy. The important consequence of Eq. (3.10) is that coupling of the matter fields with the curvature gives a nonzero contribution ΔS to the Bekenstein-Hawking entropy. In quantum theory ΔS becomes an average of the corresponding field operator on Σ.

Let us now consider the vector model of induced gravity. According to Eq. (2.4) the effective action of the theory can be written as a path integral

$$\exp i\Gamma[g]=\int[D\Phi]\exp(iI[g,\Phi]), \tag{3.11}$$

$$I[g,\Phi]\equiv\sum_{i=1}^{N}(I_s[\phi_i]+I_d[\psi_i]+\tilde{I}_v[A_i]+I_s[\varphi_i]), \tag{3.12}$$

where $\Phi=\{\phi_i,\psi_i,A_i,\varphi_i\}$. The functionals I_s, I_d, and $\tilde{I}_v$ are defined by Eqs. (2.1), (2.2), and (2.9), respectively. The origin of the scalar fields φ_i in Eq. (3.11) is related to the quantization of the massive vector fields A_i. It is assumed that φ_i obey the "wrong" (Fermi) statistics in order to reproduce Eq. (2.7). As follows from Eq. (2.9), the total "classical action" $I[g,\Phi]$ includes the nonminimal couplings of the vector fields A_i. By using formula (3.10) in the theory with the action $\tilde{I}_v[A]$ one obtains the nonzero term

$$\Delta S=-8\pi\int_{\Sigma}t_{\mu}n_{\nu}t_{\lambda}n_{\rho}\frac{\partial\tilde{L}_v}{\partial R_{\mu\nu\lambda\rho}}d\sigma=\pi\int_{\Sigma}(t^{\mu}t^{\nu}-n^{\mu}n^{\nu})A_{\mu}A_{\nu}d\sigma, \tag{3.13}$$

where we put $\tilde{I}_v[A]=\int\tilde{L}_v dV$. In the induced gravity such terms result in a correction to the entropy of a black hole. To first order in the Planck constant this correction simply is

$$\Delta S=\pi\int_{\Sigma}(t^{\mu}t^{\nu}-n^{\mu}n^{\nu})\sum_{i=1}^{N}\langle\hat{A}_{i\mu}\hat{A}_{i\nu}\rangle d\sigma\equiv-Q. \tag{3.14}$$

Here the average $\langle\hat{A}_{i\mu}\hat{A}_{i\nu}\rangle$ is understood as a regularized quantity. The quantity Q has the meaning of Wald's Noether charge associated with nonminimal interaction terms of the vector field. The sign minus on the right-hand side (RHS) of Eq. (3.14) is chosen so that Q is positive.

By using the Pauli-Villars regularization one finds that, in the Rindler approximation,

$$\langle\hat{A}_{i\mu}\hat{A}_{i\nu}\rangle=\eta_{\mu\nu}\frac{b(m_{v,i})}{16\pi^2}, \tag{3.15}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and function $b(m_{v,i})$ is defined by Eq. (3.3). Equation (3.15) gives

$$Q=\frac{1}{8\pi}\sum_{i=1}^{N}b(m_i)\mathcal{A}=\frac{1}{8\pi}\left(cN\mu^2-\ln\mu^2\sum_{i=1}^{N}m_{v,i}^2+\sum_{i=1}^{N}m_{v,i}^2\ln m_{v,i}^2\right)\mathcal{A}. \tag{3.16}$$

This result allows one to show that the total Bekenstein-Hawking entropy S^{BH} in induced gravity is the difference of statistical-mechanical entropy S^{SM} [see Eq. (3.9)] and the Noether charge Q. As can be easily seen, the divergences of S^{SM} are exactly canceled by the divergences of the charge Q, so that one gets the finite expression

$$S^{SM}-Q=\frac{1}{48\pi}\sum_{i=1}^{N}[m_{s,i}^2\ln m_{s,i}^2+2m_{d,i}^2\ln m_{d,i}^2-3m_{v,i}^2\ln m_{v,i}^2]\mathcal{A}=\frac{\mathcal{A}}{4G}=S^{BH}. \tag{3.17}$$

This expression coincides exactly with the Bekenstein-Hawking entropy in induced gravity where the induced Newton constant is determined by formula (2.20).

As was argued in Ref. [10], the statistical-mechanical reason why the Noether charge appears in Eq. (3.17) is related to the fact that the canonical Hamiltonian H and the energy E of the system are different. H defines the free energy (3.1) and entropy S^{SM} while the energy E is connected with the spectrum of the mass of the black hole. In the Appendix we show that, for the vector model,

$$H-E=\frac{\kappa}{2\pi}Q. \tag{3.18}$$

This is the same relation which was found in [10] for the induced gravity model with nonminimally coupled scalar fields. Relation (3.18) can be used to provide the statistical-mechanical interpretation of the subtraction of the charge Q in the black hole entropy formula (3.17). This interpretation repeats the one already given in [10]: subtraction of Q is needed in order to pass from the distribution over the energy in canonical ensemble of constituent fields to the distribution over the black hole mass in the black hole canonical ensemble which determines S^{BH}.

IV. BLACK HOLE ENTROPY AND 2D QUANTUM THEORY ON Σ

Our aim now is to relate the Bekenstein-Hawking entropy S^{BH}, Eq. (3.17), to a 2D quantum theory of free massive fields ''living'' on the bifurcation surface Σ of the horizon. To this aim it is instructive to represent expression (3.17) in another equivalent form. First, let us note that in the Rindler approximation the regularized correlators of the scalar, spinor, and vector fields of the mass m have the simple form

$$\langle\hat{\phi}^2\rangle=\frac{b(m)}{16\pi^2},\quad \langle\hat{\bar{\psi}}\hat{\psi}\rangle=4m\langle\hat{\phi}^2\rangle,\quad \langle\hat{V}_\mu\hat{V}^\mu\rangle=3\langle\hat{\phi}^2\rangle. \tag{4.1}$$

In the Pauli-Villars regularization the function b is defined by Eq. (3.3). From Eqs. (3.17) and (4.1) we easily find that

$$S^{BH}=\frac{\pi}{6}\sum_{i=1}^{N}\int_\Sigma d\sigma\left[2\langle\hat{\phi}_i^2\rangle+\frac{1}{m_{d,i}}\langle\hat{\bar{\psi}}_i\hat{\psi}_i\rangle-2\langle\hat{V}_i^2\rangle\right]. \tag{4.2}$$

One can check that the divergences in correlators in Eq. (4.2) are canceled because of induced gravity constraint $q(1)=0$; see Eqs. (2.15) and (2.17). Since the surface Σ of bifurcation of horizons is a set of fixed points of the Killing vector, only zero-frequency (''soft'') modes contribute to the correlators on Σ (for a detailed discussion of this point, see [10]).

As was shown in [10], the correlator of scalar fields taken on the bifurcation surface of the Killing horizons behaves effectively as a two-dimensional operator. Namely, if z and z' are the coordinates of the points x and x' on Σ [see Eq. (3.4)], then

$$\langle\hat{\phi}(x(z))\hat{\phi}(x(z'))\rangle=-\frac{1}{4\pi}\langle z|\ln O_\Sigma|z'\rangle, \tag{4.3}$$

$$O_\Sigma=-\nabla_\Sigma^2+m^2, \tag{4.4}$$

where $-\nabla_\Sigma^2$ is the Laplacian on Σ. The left and right parts of Eq. (4.3) should be calculated in the same regularization. It should be emphasized that Eq. (4.3) is an exact relation for Rindler space.[5] For the RHS of Eq. (4.3) we find that

$$-\frac{1}{4\pi}\int_\Sigma\langle z|\ln O_\Sigma|z\rangle d\sigma=-\frac{1}{4\pi}\ln\det(-\nabla_\Sigma^2+m^2)$$
$$\equiv-\frac{1}{2\pi}W_s(m). \tag{4.5}$$

The functional $W_s(m)$ has the meaning of the effective action of a 2D quantum field χ given on Σ. It can be expressed in terms of the Euclidean path integral as

$$e^{-W_s(m)}=\int D[\chi]\exp\left[-\frac{1}{2}\int_\Sigma[(\nabla_\Sigma\chi)^2+m^2\chi^2]d\sigma\right], \tag{4.6}$$

where $D[\chi]$ is a covariant measure. In Ref. [10] a two-dimensional auxiliary field χ ''living'' on the bifurcation surface Σ was called a *flucton* field to distinguish it from the 4D fields in the black hole exterior. From Eqs. (4.3) and (4.5) one obtains

$$\int_\Sigma d\sigma\langle\hat{\phi}^2(x(z))\rangle=-\frac{1}{2\pi}W_s(m). \tag{4.7}$$

It follows from Eqs. (4.2) and (4.7) that the contribution of scalar fields to the black hole entropy S^{BH} can be interpreted in terms of a 2D quantum theory of fluctons on Σ.

We now find an analogous representation for the contribution to S^{BH} from spinor and vector fields. The correlators of these fields in the coinciding points are tensors in certain representations of the Lorentz group. Different parts of these tensors have different two-dimensional interpretations. We begin with the correlator (4.8) of vector fields restricted on Σ:

$$\langle\hat{V}_\mu(x(z))\hat{V}_\nu(x(z'))\rangle=\langle\hat{A}_\mu(x(z))\hat{A}_\nu(x(z'))\rangle$$
$$+m^{-2}\nabla_\mu\nabla'_\nu\langle\hat{\varphi}(x(z))\hat{\varphi}(x(z'))\rangle, \tag{4.8}$$

where $\hat{\varphi}$ is a scalar field of the same mass as $\hat{V}_\mu$. Let us consider the components of tensor quantities in the Minkowski coordinates X^m. With respect to the coordinate transformations on Σ, components of a tensor with indices 0 and 3 behave as scalars while components with indices 1 and 2 transform as vectors on Σ. By using the arguments of Ref.

[5]It can be generalized to curved backgrounds with a Killing horizon. In the general case the operator O_Σ for very massive fields can be found by comparing Schwinger-DeWitt asymptotics of four- and two-dimensional operators; for details see [10]. The key property which allows a two-dimensional interpretation of the 4D correlators on Σ is that Σ is a geodesic surface. That is, any 4D geodesic which begins and ends on Σ coincides with the 2D geodesic on Σ.

[10] one can express the correlator $\langle \hat{A}_\mu \hat{A}_\nu \rangle$ with $\mu,\nu=1,2$ in terms of the 2D vector Laplacian on Σ. Analogously, components of the correlator with $\mu,\nu=0,3$ can be represented in terms of the 2D scalar Laplacian. Thus, we find that

$$\int_\Sigma d\sigma \langle \hat{A}^1(x(z))\hat{A}_1(x(z))+\hat{A}^2(x(z))\hat{A}_2(x(z))\rangle$$
$$=-\frac{1}{2\pi}\tilde{W}_v(m), \tag{4.9}$$

$$\int_\Sigma d\sigma \langle \hat{A}^0(x(z))\hat{A}_0(x(z))+\hat{A}^3(x(z))\hat{A}_3(x(z))\rangle$$
$$=-2\frac{1}{2\pi}W_s(m). \tag{4.10}$$

W_s is the effective action of a scalar field on Σ and $\tilde{W}_v(m)$ is the effective action of a vector field on Σ with the same mass m as that of 4D field:

$$\tilde{W}_v(m)=\frac{1}{2}\log\det\left[\left(-\nabla^2_\Sigma+\frac{1}{2}R_\Sigma+m^2\right)\delta^A_B\right]. \tag{4.11}$$

Here $A,B=1,2$ and R_Σ is the curvature of Σ which can be neglected in the Rindler approximation. It follows from Eqs. (4.8)–(4.10) that

$$\int_\Sigma d\sigma \langle \hat{V}^\mu(x(z))\hat{V}_\mu(x(z))\rangle$$
$$=\int_\Sigma d\sigma \langle \hat{A}^\mu(x(z))\hat{A}_\mu(x(z))-\hat{\varphi}(x(z))\hat{\varphi}(x(z))\rangle$$
$$=-\frac{1}{2\pi}[W_v(m)+2W_s(m)], \tag{4.12}$$

$$W_v(m)=\tilde{W}_v(m)-W_s(m). \tag{4.13}$$

The functional $W_v(m)$ corresponds to the quantization of the massive 2D vector field described by the classical action analogous to the 4D action (2.3).

Similar relations can be obtained for spinor fields. One 4D Dirac spinor corresponds to two 2D Dirac spinors on Σ. One easily finds that

$$\int_\Sigma d\sigma \langle \hat{\bar{\psi}}(x(z))\hat{\psi}(x(z))\rangle=\frac{m}{\pi}W_d(m), \tag{4.14}$$

where $W_d(m)$ is the effective action of 2D spinors on Σ with mass m.

By using Eqs. (4.7), (4.12), and (4.14) in expression (4.2) for the Bekenstein-Hawking entropy in induced gravity we find

$$S^{BH}=\frac{1}{6}\sum_{i=1}^{N}[-W_s(m_{s,i})+W_d(m_{d,i})$$
$$+W_v(m_{v,i})+2W_s(m_{v,i})]. \tag{4.15}$$

This form of the entropy looks similar to the effective action of a two-dimensional quantum field model on the surface Σ. To make this similarity more evident let us consider the concrete induced gravity model with partially broken supersymmetry which was discussed in Sec. II. In this model the masses of vector and spinor fields coincide, $m_{v,i}=m_{d,i}=m_i$. As a result, $W_v(m)=W_s(m)=-\frac{1}{2}W_d(m)$ and Eq. (4.15) takes the form

$$S^{BH}=-\frac{1}{12}\sum_{i=1}^{N}[2W_s(m_{s,i})+W_d(m_i)]\equiv-\frac{1}{12}\Gamma^{(2)}. \tag{4.16}$$

The quantity $\Gamma^{(2)}$ is the effective action of a 2D model which consists of N spinor fields with masses m_i and $2N$ scalar fields with masses $m_{s,i}$.

In fact, we have 2D induced gravity on Σ. The condition that the 4D curvature be small compared to the masses of the fields guarantees that the two-dimensional curvature of Σ is small as well. So the 2D effective action $\Gamma^{(2)}$ can be computed as an expansion in curvature. The leading term in this expansion is the cosmological constant term

$$\Gamma^{(2)}[\gamma]\simeq\int_\Sigma\sqrt{\gamma}\,d^2x\,\lambda. \tag{4.17}$$

Here λ is the ''induced'' 2D cosmological constant which is expressed in terms of the induced 4D Newton constant (2.20) as $\lambda=-3/G$. The constraints which provide the ultraviolet finiteness of the 4D Newton constant [see Eqs. (2.17)] automatically guarantee the finiteness of 2D cosmological constant.

The 2D model described by functional $\Gamma^{(2)}$ can be obtained from the supersymmetric model with N multiplets consisting of a spinor and two scalar fields. The split of the masses of spinor and scalar fields breaks the supersymmetry and yields a nonvanishing 2D cosmological constant.

Of course, the suggested connection between 4D and 2D theories is not unique, and one may expect that in general the coefficient on the RHS of Eq. (4.16) can be another rational number. Let us emphasize that the considered models of induced gravity are phenomenological and admit a large arbitrariness in the choice of masses of the constituent fields. One may hope that if the induced gravity is obtained from an underlying fundamental theory, the masses of the fields will be fixed by some principle which will determine the coefficient in Eq. (4.16).

A remark is also in order about the two-dimensional interpretation of the Noether charge Q. By taking into account Eq. (3.16) it is easy to show that, in the Rindler approximation,

$$Q=-\sum_i W_v(m_{v,i}). \tag{4.18}$$

This relation holds in any induced gravity model with vector fields and does not require additional conditions on the masses of the constituent fields. It enables one to relate Q to a quantum theory of 2D vector fields on Σ.

V. DISCUSSION

To summarize, we considered a class of induced gravity models where the low-energy gravitational field is generated by quantum one-loop effects in a system of heavy constituents. The vector models presented here consist of massive scalar, spinor, and vector constituent fields, and do not require nonminimal couplings of the scalar constituents. We demonstrated that the general mechanism of the entropy generation in the induced gravity proposed in Ref. [10] does work, and that the Bekenstein-Hawking entropy can be derived by a statistical-mechanical counting of the energy states of heavy constituents.

It was further demonstrated that the expression for the Bekenstein-Hawking entropy in the induced gravity can be identically rewritten in terms of fluctuations of the constituent fields at the event horizon. The latter are determined only by zero-frequency (''soft'') solutions of the corresponding field equations. These ''soft'' modes are uniquely defined by their asymptotics at the bifurcation sphere of horizons Σ. Using this property, it was explicitly demonstrated that the fluctuations of the constituent fields at the horizon coincide with the effective action of two-dimensional (flucton) fields on Σ. This mechanism is somewhat similar to the idea of the holography [15–21]. We hope to discuss this relation in more details somewhere else.

As a result of the two-dimensional reduction, the Bekenstein-Hawking entropy appears to be equal to $|\lambda|\mathcal{A}/12$, where λ is the 2D cosmological constant induced on the surface of the horizon by 2D flucton fields. This implies that the degrees of freedom responsible for the black hole entropy in the induced gravity can be related to surface degrees of freedom of the black hole horizon. Such a conclusion is supported by the observation that since the masses of the constituents are very high (of the order of the Planckian mass), the fluctuations of the constituent fields near the horizon can be directly connected with the fluctuations of the 2D geometry of the horizon. This might bring a connection with the well-known results of statistical computations of black hole entropy of 3D black holes [4,5].

It should be emphasized, once again, that the induced gravity approach does not pretend to explain the black hole entropy from first principles of the fundamental theory of quantum gravity (such as the string theory) but it allows one to demonstrate the universality of the entropy and its independence of the concrete details of such a theory. It gives us a hint that only a few quite general properties of the fundamental theory (such as the low-energy gravity as induced phenomenon, finiteness of the low-energy coupling constants, holography, and so on) are really required for a statistical-mechanical explanation of the black hole entropy.

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APPENDIX: ENERGY, HAMILTONIAN, AND NOETHER CHARGE FOR VECTOR FIELDS

Here we consider the relation between the energy and the Hamiltonian for the vector model and prove Eq. (3.18). Let us recall that the classical energy E of a field Φ in a 3D region $\mathcal{B}$ is defined by the stress-energy tensor

$$E=\int_{\mathcal{B}} T_{\mu\nu}\zeta^{\mu}d\sigma^{\nu}, \quad T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta I[\Phi]}{\delta g^{\mu\nu}}, \tag{A1}$$

where $d\sigma^{\nu}$ is the future directed vector of the volume element on $\mathcal{B}$, ζ^{μ} is the timelike Killing vector, and $I[\Phi]$ is the classical action of Φ. The canonical energy is

$$H=\int_{\mathcal{B}}\left(\frac{\partial L(\Phi)}{\partial\nabla^{\nu}\Phi}\mathcal{L}_{\zeta}\Phi-\zeta_{\nu}L(\Phi)\right)d\sigma^{\nu}, \tag{A2}$$

where $\mathcal{L}_{\zeta}$ is the Lie derivative along ζ^{μ} and $L(\Phi)$ is the Lagrangian of the field ($I[\Phi]=\int dVL(\Phi)$).

In the quantum theory we are dealing with quantum averages of the energy and the canonical energy. For the theory under consideration, Eq. (3.11), one has

$$\overline{E}=\sum_{i=1}^{N}(\overline{E}_{s,i}+\overline{E}_{d,i}+\overline{\tilde{E}}_{v,i}-\overline{E'}_{s,i}), \tag{A3}$$

$$\overline{H}=\sum_{i=1}^{N}(\overline{H}_{s,i}+\overline{H}_{d,i}+\overline{\tilde{H}}_{v,i}-\overline{H'}_{s,i}). \tag{A4}$$

In these relations the average $\overline{C}$ of $\hat{C}$ is

$$\overline{C}=e^{-i\Gamma[g]}\int[D\Phi]C[\Phi,g]e^{iI[g,\Phi]}, \tag{A5}$$

and $\overline{E_i}$ and $\overline{H_i}$ are the energy and the canonical energy of each of the fields which enter total action (3.12). The quantities $\overline{\tilde{E}}_{v,i}$, $\overline{\tilde{H}}_{v,i}$ and $\overline{E'}_{s,i}$, $\overline{H'}_{s,i}$ correspond to the fields A_i^{μ} and φ_i, respectively. These fields appear under quantization of the vector constituents V_i^{μ}. The minus sign by $\overline{E'}_{s,i}$ and $\overline{H'}_{s,i}$ is the result of the ''wrong'' statistics of the fields φ_i.[6]

It can be easily shown that the energy and the canonical energy coincide if the action I does not contain the terms with curvature. The only fields which explicitly contain the curvature term are vector fields A_i^{μ}. For this reason one has

[6]This result can be obtained directly if one starts with the expression for the energy and canonical energy for a vector field V_{μ}, rewrite them in the point-split form, and take into account that

$$\langle\hat{V}_{\mu}(x)\hat{V}_{\nu}(x')\rangle=\langle\hat{A}_{\mu}(x)\hat{A}_{\nu}(x')\rangle+m^{-2}\nabla_{\mu}\nabla'_{\nu}\langle\hat{\varphi}(x)\hat{\varphi}(x')\rangle.$$

$$\overline{H}-\overline{E}=\sum_{i=1}^{N}(\overline{\widetilde{H}}_{v,i}-\overline{\widetilde{E}}_{v,i}). \tag{A6}$$

Let us discuss first the difference between the energy $\widetilde{E}_v$ and the Hamiltonian $\widetilde{H}_v$ of a classical vector field A_μ described by action $\widetilde{I}_v[A]$; see Eq. (2.9). $\widetilde{E}_v$ and $\widetilde{H}_v$ are obtained from $\widetilde{I}_v[A]$ by formulas (A1) and (A2). The difference between these quantities appears because of the variation over the metric of the curvature coupling term in $\widetilde{I}_v[A]$, and so one has

$$\widetilde{H}_v-\widetilde{E}_v=2\int_{\mathcal{B}}d\sigma_\nu\zeta_\mu\nabla_\rho\nabla_\sigma\left(\frac{\partial\widetilde{L}_v}{\partial R_{\mu\sigma\rho\nu}}+\frac{\partial\widetilde{L}_v}{\partial R_{\nu\sigma\rho\mu}}\right). \tag{A7}$$

In the Rindler approximation the integral can be transformed to the total divergence

$$\widetilde{H}_v-\widetilde{E}_v=2\int_{\mathcal{B}}d\sigma_\nu\nabla_\rho\times\left[(-\zeta_{\mu;\sigma}+\zeta_\mu\nabla_\sigma)\left(\frac{\partial\widetilde{L}_v}{\partial R_{\mu\sigma\rho\nu}}+\frac{\partial\widetilde{L}_v}{\partial R_{\nu\sigma\rho\mu}}\right)\right]. \tag{A8}$$

When the region $\mathcal{B}$ is the region of the black hole exterior, the black hole horizon is one of its boundaries. Then the integral on the RHS of Eq. (A8) is reduced to two terms: one term comes from the spatial boundary of $\mathcal{B}$ and the other one from the bifurcation surface Σ. The terms on the spatial boundary can be eliminated by the proper choice of boundary conditions. However, the term on Σ cannot be eliminated. By taking into account that $\zeta_\mu=0$ and $\zeta_{\mu;\sigma}=\kappa(t_\mu n_\sigma-t_\sigma n_\mu)$ on Σ one obtains, from Eq. (A8),

$$\widetilde{H}_v-\widetilde{E}_v=4\kappa\int_\Sigma t_\mu n_\nu t_\lambda n_\rho\frac{\partial\widetilde{L}_v}{\partial R_{\mu\nu\lambda\rho}}d\sigma. \tag{A9}$$

Derivation of the analogous relation for general diffeomorphism invariant theories is given in [29]. By summing over all the vector fields $A_{i,\mu}$ which enter the model, using Eq. (A6), and comparing this with Eqs. (3.13) and (3.14) we see that, for the vector induced gravity model,

$$H-E=\frac{\kappa}{2\pi}Q, \tag{A10}$$

where Q is the Noether charge. To obtain from Eq. (A10) the result in quantum theory one has to replace the quantities in this formula by the corresponding quantum averages.

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